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TECHNICAL NOTES

Pressure perturbations method for analysis of transient compressible gas flow around wells in porous media

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1. INTRODUCTION

In the last two decades, a new power generating technology emerged, involving Compressed Air Energy Storage (CAES). Air compressors using off-peak electric energy supplied by base load power plants charge CAES underground reservoirs. During peak load periods, air released from such reservoirs expands in turbines, generating electric power [1]. Candidate reservoirs are excavated hard rock, solution mined salt caverns, abandoned mines, depleted gas reservoirs and aquifers. The latter two reservoirs contain a subterranean water bearing (aquiferous) porous rock, from which the water is displaced by compressed air.

Transient compressible gas flow around wells in porous media arises, e.g. in conjunction with CAES and gas storage in aquifers and in other underground porous reservoirs [2]. While in seasonal gas storage the pressure varies relatively slowly, and the respective flow can be considered as quasi incompressible, in the case of CAES and of peak shaving gas storage, pressure changes are relatively big and of relatively small duration. Thus, in the latter case one must fully account for the gas compressibility. Based on Darcy's law, transient compressible gas flow in porous media is described by a nonlinear diffusion type partial differential equation.

A self-similar solution to such types of problems for a single well located in an infinite reservoir is known [3]. For this solution, a zero well radius, constant well flow rate, and uniform initial pressure distribution are assumed. However, in many cases of practical significance we can not neglect the well radius and assume a reservoir of infinite extent. For such cases, similarity solutions do not exist, and we must apply either numerical or approximate analytical methods.

One such example is a study performed to investigate the behavior and suitability of an aquifer-based compressed-air energy storage plant [2]. There, solutions for the air pressure distribution around a well were obtained by using a finite difference scheme.

In the present work, an analytical method is developed that fully accounts for geometric and fluid flow factors, such as finite dimensions of the reservoir and non-zero well radius, time dependent boundary conditions, and an arbitrary initial pressure distribution.

2. PRESSURE PERTURBATIONS METHOD

The general momentum equation for fluid flow in porous media is given by [4]

$$\nabla p = -\frac{\mu}{k}\vec{v}+\rho\vec{g}-\frac{\rho}{s_p}\frac{D\vec{v}}{Dt}+\frac{\mu}{s_p}\nabla^2\vec{v}-\frac{\rho C}{k^{1/2}}|\vec{v}|\vec{v}. \quad (1)$$

In order to assess the importance of the various terms in (1) for our case, let us introduce the typical reservoir data: $\mu = 1.85\times10^{-5}$ kg m s^{-1}, $k = 1\times10^{-12}$ m^2, $\overline{\Delta p} = 20\times10^5$ Pa, $\bar{p} = 50\times10^5$ Pa, $l_c = 100$ m, $s_p = 0.06$ and $t_d = 2\times10^4$ s. Based on these entities, an order of magnitude analysis shows that the ratios of the third, fourth and fifth terms on the RHS of (1) to the first term are 10^{-9}, 10^{-15} and 10^{-3}, respectively. Thus, the third, fourth and fifth terms on the RHS of (1) may be neglected. Since the gas movement in the reservoir is mainly horizontal, the second term on the RHS of (1) may also be neglected. It leads to the well known classical Darcy's law equation [3].

Based on Darcy's law, the pressure distribution in transient isothermal ideal gas flow through porous media for constant thermophysical and geometric properties is described by the classical nonlinear diffusion equation [5]–[7]

$$\frac{\partial p}{\partial t} = \frac{k}{2\mu s_p}\nabla^2 p^2 \quad (2)$$

or equivalently

$$\frac{\mu s_p}{k}\frac{\partial(\ln p)}{\partial t} = \nabla^2 p+\frac{1}{p}(\nabla p)^2 \quad (3)$$

with suitable boundary and initial conditions.

Analytical solutions to equation (2) are limited to particular cases. Generally, equation (2) is solved either numerically, or by using approximate analytical methods, one of which is described in the sequel.

Assume the gas pressure as a sum of two components

$$p(\vec{r},t) = p_e(t)+p_D(\vec{r},t) \quad (4)$$

where $\vec{r}$ is a position vector.

In equation (4), $p_e(t)$ is a volume averaged gas pressure in the reservoir given by

$$p_e(t) = \frac{1}{V}\int_V p(\vec{r},t)\,dV \quad (5)$$

where V is the gas volume in the reservoir and p_D designates the pressure perturbation due to a finite permeability of the porous reservoir. From Darcy's law and from equation (4) it turns out that

$$\nabla p_D = \nabla p = -\frac{\mu}{k}\vec{v} \quad (6)$$

where $\vec{v}$ is the superficial velocity.

From formulae (5) and (4) follow two things: that the space averaged instantaneous pressure perturbation is equal to zero, and that p_e defines the mass of the gas in the reservoir, namely

NOMENCLATURE

- $a(t) = (k/\mu s_p) p_e(t)$
- $B(r)$ an initial condition
- $C = 0.143 s_p^{-1.5}$
- g gravitational acceleration
- $F(t)$ boundary condition
- $H(r)$ an auxiliary function defined in equation (18)
- k permeability
- l_c characteristic length of a reservoir
- M gas mass in a reservoir
- $\dot{m}_g$ gas mass flow rate
- p, p_e, p_D local pressure, average gas pressure and pressure perturbation, respectively
- $Q(r, t)$ source function defined in equation (20)
- $Q_n(t)$ coefficients defined in equation (27)
- R dimensionless radial coordinate scaled with respect to r_w
- $R_n(r)$ eigenfunction
- R_g gas constant
- $\mathbf{r}$ position vector
- r radial coordinate
- s_p porosity
- T temperature
- t time
- $\mathbf{v}$ superficial gas velocity.

Greek symbols

- $\Gamma_n(t)$ coefficient of expanding $\phi(r, t)$ in a series of eigenfunctions
- ε average dimensionless value of pressure perturbation defined in equation (10)
- λ_n, Λ_n dimensional and dimensionless eigenvalue, respectively
- μ gas dynamic viscosity
- $\Phi(r, t)$ a potential function defined in equation (41)
- $\Phi_R = (2/r_w^2)\Phi$ reduced potential
- $\phi(r, t)$ function defined in equation (17)
- ρ gas density.

Subscripts

- cn characteristic
- d duration
- e external
- w well.

$$\bar{p}_D = \frac{1}{V}\int_V p_D(\vec{r}, t)\, dV = 0 \tag{7}$$

for any gas, while for an ideal gas we have

$$M(t) = \frac{p_e(t) s_p V(t)}{R_g T} \tag{8}$$

where R_g is the gas constant and T is the gas temperature.

Therefore, within the reservoir space domain, there always exists a surface on which $p_D = 0$. This surface divides between a region of positive p_D, and a region of negative p_D.

Equation (3) may be recast in the following form:

$$\frac{\mu s_p}{k}\frac{\partial[\ln(p_e + p_D)]}{\partial t} = \nabla^2 p_D + \frac{1}{p}(\nabla p_D)^2. \tag{9}$$

From equation (9), an average dimensionless value of pressure perturbation is defined as

$$\varepsilon \approx \frac{|\overline{p_D}|}{\bar{p}} = \frac{\mu s_p l_c^2 \overline{\Delta p}}{k t_d \bar{p}^2}. \tag{10}$$

For the typical reservoir data, we obtain $\varepsilon \approx 0.04$. Thus, the pressure perturbation is small $|p_D| \ll p_e$, and equation (9) can be linearized to give

$$\frac{\partial p_D}{\partial t} = \frac{k p_e}{\mu s_p}\nabla^2 p_D - \frac{dp_e}{dt}. \tag{11}$$

Now the determination of compressible gas flow characteristics becomes simpler. The pressure perturbation p_D is defined as a solution of a linear partial differential equation (11) with proper boundary and initial conditions, since an average gas pressure p_e may be determined from an ordinary differential equation that stems from the equation (8)

$$V\frac{dp_e}{dt} + p_e\frac{dV}{dt} = \frac{R_g T}{s_p}\dot{m}_g \tag{12}$$

where $\dot{m}_g$ is the mass flow rate of gas through the boundaries of the reservoir and through the wells. From equation (6), the superficial velocity $\vec{v}$ can be determined.

3. PRESSURE PERTURBATIONS FOR A FULLY PENETRATING WELL IN A CYLINDRICAL CLOSED RESERVOIR

A model case of a fully penetrating well with radius r_w in a cylindrical closed reservoir having an outer radius r_e is considered. We assume arbitrary initial pressure distribution $p_D(r, 0) = B(r)$, a pressure gradient $\partial p_D/\partial r\,(r_w, t) = F(t)$ at the well radius and $\partial p_D/\partial r\,(r_e, t) = 0$ at the outer domain radius.

This problem will be solved by eigenfunction expansions. To this end, the boundary conditions are first homogenized by introducing a new dependent variable, which leads to the following problem formulation

$$\frac{\partial \phi}{\partial t} = \frac{a(t)}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + Q(r, t) \text{ in } r_w < r < r_e \text{ for } t > 0 \tag{13}$$

$$\phi(r, 0) = B(r) - H(r)F(0) \tag{14}$$

$$\frac{\partial \phi}{\partial r}(r_w, t) = 0 \tag{15}$$

$$\frac{\partial \phi}{\partial r}(r_e, t) = 0. \tag{16}$$

The following relations define the various variables which appear in equations (13)–(16)

$$\phi(r, t) = p_D(r, t) - H(r)F(t) \tag{17}$$

$$H(r) = \frac{r^2}{2(r_w - r_e)} + \frac{r_e r}{r_e - r_w} \tag{18}$$

$$a(t) = \frac{k}{\mu s_p} p_e(t) \tag{19}$$

$$Q(r, t) = \frac{dp_e}{dt} + a(t)F(t)\frac{r_e/r - 2}{r_e - r_w} - H(r)\frac{dF}{dt}. \tag{20}$$

The related eigenvalue problem is

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R_n}{\mathrm{d}r}\right) = -\lambda_n^2 R_n \quad (21)$$

$$\frac{\mathrm{d}R_n}{\mathrm{d}r}(r_w) = 0 \quad (22)$$

$$\frac{\mathrm{d}R_n}{\mathrm{d}r}(r_e) = 0 \quad (23)$$

which results in the following eigenfunctions (but for a constant multiplier)

$$R_n(r) = Y_1(\lambda_n r_w)J_0(\lambda_n r) - J_1(\lambda_n r_w)Y_0(\lambda_n r) \quad (24)$$

where the eigenvalues are obtained as the positive roots of

$$Y_1(\lambda_n r_w)J_1(\lambda_n r_e) - J_1(\lambda_n r_w)Y_1(\lambda_n r_e) = 0. \quad (25)$$

Next expand the source term in a series of the eigenfunctions

$$Q(r,t) = \sum_{n=1}^{\infty} Q_n(t)R_n(r) \quad (26)$$

where

$$Q_n(t) = \int_{r_w}^{r_e} Q(r,t)R_n(r)r\,\mathrm{d}r \Big/ \int_{r_w}^{r_e} R_n^2(r)r\,\mathrm{d}r, \quad n = 1,2,\ldots \quad (27)$$

Substituting $\phi(r,t) = \Sigma_{n=1}^{\infty} R_n(r)\Gamma_n(t)$ and $Q(r,t) = \Sigma_{n=1}^{\infty} Q_n(t)R_n(r)$ into equation (13) yields

$$\sum_{n=1}^{\infty} R_n\frac{\mathrm{d}\Gamma_n}{\mathrm{d}t} = \sum_{n=1}^{\infty}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R_n}{\mathrm{d}r}\right)a(t)\Gamma_n + \sum_{n=1}^{\infty} Q_n(t)R_n. \quad (28)$$

Utilizing equation (21) in equation (28) and rearranging gives

$$\sum_{n=1}^{\infty} R_n\left[\frac{\mathrm{d}\Gamma_n}{\mathrm{d}t} + a(t)\lambda_n^2\Gamma_n - Q_n(t)\right] = 0. \quad (29)$$

Since, in general, R_n are not zero, we must have

$$\frac{\mathrm{d}\Gamma_n}{\mathrm{d}t} + a(t)\lambda_n^2\Gamma_n - Q_n(t) = 0 \quad (30)$$

the solution of (30) is

$$\Gamma_n(t) = \mathrm{e}^{-\gamma_n(t)}\left[\Gamma_n(0) + \int_0^t Q_n(t)\,\mathrm{e}^{\gamma_n(t)}\,\mathrm{d}t\right] \quad (31)$$

where $\gamma_n(t) = \lambda_n^2 \int_0^t a(t)\,\mathrm{d}t$. In order to determine $\Gamma_n(0)$, employ the initial condition (14)

$$\phi(r,0) = B(r) - H(r)F(0) = \sum_{n=1}^{\infty} R_n(r)\Gamma_n(0). \quad (32)$$

Upon operating on both sides of (32) with $\int_{r_w}^{r_e} R_m(r)r\,\mathrm{d}r$, and making use of the orthogonality properties of the eigenfunctions $R_n(r)$ we obtain

$$\Gamma_n(0) = \int_{r_w}^{r_e} [B(r) - H(r)F(0)]R_n(r)r\,\mathrm{d}r \Big/ \int_{r_w}^{r_e} R_n^2(r)r\,\mathrm{d}r. \quad (33)$$

4. ANALYSIS OF SOLUTION

The general solution (31) can be used to analyze the typical time behavior of the transient compressible gas flow in a porous reservoir.

Introducing $\bar{a}$ and $\bar{Q}_n$—the average values of $a(t)$ and $Q_n(t)$—respectively, during the time interval $[0, t]$, we obtain

$$\gamma_n(t) = \lambda_n^2\bar{a}t \quad (34)$$

$$\int_0^t Q_n(t)\,\mathrm{e}^{\gamma_n(t)}\,\mathrm{d}t = \frac{\bar{Q}_n}{\lambda_n^2\bar{a}}(\mathrm{e}^{\lambda_n^2\bar{a}t} - 1) \quad (35)$$

so that

$$\Gamma_n(t) = \mathrm{e}^{-\lambda_n^2\bar{a}t}\Gamma_n(0) + \frac{\bar{Q}_n}{\lambda_n^2\bar{a}}(1 - \mathrm{e}^{-\lambda_n^2\bar{a}t}). \quad (36)$$

Using the characteristic time $t_{cn} = \lambda_n^2\bar{a} = \mu s_p l_c^2/k\bar{p}_e\Lambda_n^2$, where $l_c = r_e - r_w$ is a characteristic length, and $\Lambda_n = \lambda_n l_c$ is a dimensionless eigenvalue, it is possible to rewrite equation (36) in the form

$$\Gamma_n(t) = \mathrm{e}^{-t/t_{cn}}\Gamma_n(0) + \frac{\bar{Q}_n}{\lambda_n^2\bar{a}}(1 - \mathrm{e}^{-t/t_{cn}}). \quad (37)$$

For the typical reservoir data and $r_e/r_w > 500$, values of the characteristic time are: $t_{c1} = 151$ s, $t_{c2} = 45$ s, $t_{c3} = 21$ s, $t_{c4} = 12$ s, $t_{c5} = 8$ s. These values decrease when n increases. For the present calculations, the data for Λ_n from [8] are used.

We realize that even for $n = 1$ the characteristic time is small. Comparing it with a typical value of the time interval for reservoir charge and discharge $t_d = 2\times10^4$ s, we may conclude that in engineering practice we usually have $t_d/t_{c1} > 100$. In this case, we may neglect terms including $\mathrm{e}^{-t/t_{cn}}$ in expression (37), and we are left with

$$\Gamma_n(t) = \frac{\bar{Q}_n}{\lambda_n^2\bar{a}}. \quad (38)$$

The solution (38) does not depend on initial conditions and corresponds to the case where the governing equation (11) for pressure perturbations does not include the term $\partial p_D/\partial t$. This case may be called *the stabilized gas flow regime.*

5. THE STABILIZED GAS FLOW REGIME

The previous analysis leads to the conclusion that for a general case of compressible gas flow in a porous reservoir, the initial transient period may be ignored for many practical engineering applications, and only the stabilized gas flow regime prevails. This is allowed when pressure perturbations are small and the process duration is much longer than the characteristic time.

From equation (11) it turns out that pressure perturbations for a stabilized gas flow regime are determined by a Poisson equation with appropriate boundary conditions

$$\nabla^2\Phi = 1 \quad (39)$$

where the potential Φ satisfies the condition

$$\int_V \Phi(\vec{r},t)\,\mathrm{d}V = 0. \quad (40)$$

The potential Φ in the equation (39) is defined as

$$\Phi = p_D\frac{kp_e}{\mu s_p\cdot\dfrac{\mathrm{d}p_e}{\mathrm{d}\tau}}. \quad (41)$$

Equation (39) determines a potential function of space coordinates and time. In this equation time is a parameter.

For the present case of a fully penetrating well of radius r_w in a closed reservoir with $\partial\Phi/\partial r(r_e, 0) = 0$, the solution may be represented by an expression

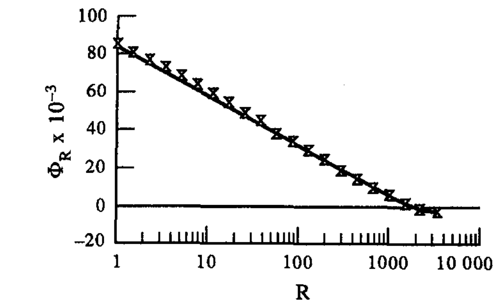

Fig. 1. Reduced potential Φ_R distribution around a well in a closed cylindrical reservoir with $R_e = 3378$ vs dimensionless radius R.

$$\Phi_R = \frac{2}{r_w^2}\Phi = \tfrac{1}{2}R^2 - R_e^2 \ln R + \frac{R_e^2(\ln R_e - 1/2) + 1/2}{1 - R_e^{-2}} - \tfrac{1}{4}(R_e^2 + 1) \quad (42)$$

where Φ_R is a reduced potential, $R = r/r_w$, and $R_e = r_e/r_w$.

Figure 1 illustrates the spatial distribution of the reduced potential Φ_R for a case with $R_e = 3378$. The curve on the graph corresponds to formula (42), and the points represent a numerical solution of the nonlinear diffusion equation (2) by [2]. We see that our approximate analytical solution is quite accurate. It is important to note that, in the analyzed case, pressure perturbations are not very small: $|p_D|/p \approx 0.2$. It means that the pressure perturbations method has a wider range of applicability then it was *a priori* assumed. As Φ_R is proportional to the instantaneous value of the pressure perturbation p_D, we realize that the region of the well influence is relatively small, $R < 300$, and in the major part of the reservoir the pressure perturbation is negligibly small. Thus, to an acceptable accuracy, instantaneous gas pressure values may be considered as $p(t) = p_e(t)$ for $R \geqslant 300$.

6. CONCLUSIONS

The mathematical description of transient compressible gas flow in porous reservoirs involves a nonlinear diffusion equation for the pressure distribution. In the present analysis, an approximate pressure perturbations method is developed. It leads to a linear diffusion equation for the pressure perturbations and to an ordinary differential equation for the space averaged instantaneous pressure.

As a model case, an analytical solution for the pressure perturbations is derived for a fully penetrating well located in a cylindrical closed reservoir. It is assumed that the well gas flow rate is an arbitrary function of time and likewise is the initial pressure distribution. Analysis of the solution shows that after a short initial period, the stabilized gas flow regime starts. In this regime, the flow characteristics do not depend on the initial conditions.

In the stabilized gas flow regime, the governing equation is of the Poisson type for potential functions of space and time that include time as a parameter. A solution of this equation for a fully penetrating well is derived.

Comparison with data obtained by a numerical solution of the nonlinear diffusion equation demonstrates that the pressure perturbations method ensures a satisfactory accuracy of the analysis.

A big variety of problems concerning transient compressible gas flow in porous media can be solved using the pressure perturbations method. Among these are a partially penetrating well, a multiple well case, etc.

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